

Non-Universal Gaugino Masses and Natural Supersymmetry

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Abstract

We demonstrate that natural supersymmetry is readily realized in the framework of $SU(4)_c \times SU(2)_L \times SU(2)_R$ with non-universal gaugino masses. Focusing on ameliorating the little hierarchy problem, we explore the parameter space of this model which yields small fine-tuning measuring parameters (natural supersymmetry) at the electroweak scale (Δ_{EW}) as well as at high scale (Δ_{HS}). It is possible to have both Δ_{EW} and Δ_{HS} less than 100 in these models, (2% or better fine-tuning), while keeping the light CP-even (Standard Model-like) Higgs mass in the 123 GeV-127 GeV range. The light stop quark mass lies in the range $700 \text{ GeV} < m_{\tilde{t}_1} < 1500 \text{ GeV}$, and the range for the light stau lepton mass is $900 \text{ GeV} < m_{\tilde{\tau}_1} < 1300 \text{ GeV}$. The first two family squarks are in the mass range $3000 \text{ GeV} < m_{\tilde{t}_1} < 4500 \text{ GeV}$, and for the gluino we find $2500 \text{ GeV} < m_{\tilde{g}_1} < 3500 \text{ GeV}$. We do not find any solution with natural supersymmetry which yields significant enhancement for Higgs production and decay in the diphoton channel.

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1 Introduction

The ATLAS and CMS collaborations at the Large Hadron Collider (LHC) have independently reported the discovery [1, 2] of a particle with production and decay modes that appear more or less consistent with the Standard Model (SM) Higgs boson of mass $m_h \sim 125$ GeV. The Minimal Supersymmetric Standard Model (MSSM) can accommodate values of $m_h \sim 125$ GeV, but this requires either a very large, $\mathcal{O}(\text{few} - 10)$ TeV, stop quark mass [3], or a large soft supersymmetry breaking (SSB) trilinear A_t -term, with a stop quark mass of around a TeV [4]. Such a heavy stop quark leads to the so-called “little hierarchy” (or “natural supersymmetry”) problem [5] because, in implementing radiative electroweak symmetry breaking, TeV scale quantities must conspire to yield the (100 GeV) electroweak mass scale (M_{EW}). Discussion of natural supersymmetry in light of a 125 GeV Higgs boson are found in Ref. [6, 7, 8].

In ref. [7] it was argued that in order to satisfactorily resolve the little hierarchy problem in a supersymmetric theory, one should investigate the fine-tuning condition not only at M_{EW} scale but also at some high, presumably GUT scale, too. It was found [7] that in the Constrained MSSM (CMSSM) case, the GUT scale fine-tuning condition for the little hierarchy problem is at least ten times more stringent compared to the same condition at M_{EW} . In this paper we adopt their procedure for analyzing the naturalness problem in our class of supersymmetric models.

It has been shown in [8, 9, 10] that non-universal gaugino masses at M_{GUT} can help resolve the little hierarchy problem. To show this, the authors have studied a variety of gaugino mass ratios obtained from some underlying theories. In this paper we revisit the study performed in ref. [10] based on the $SU(4)_c \times SU(2)_L \times SU(2)_R$ (4-2-2) gauge supersymmetry [11], which provides a natural setup for non-universal gaugino masses. In this paper, we perform a more elaborate study by employing the ISAJET 7.84 package [12] which can be used to implement the various phenomenological and cosmological constraints. We show that the little hierarchy problem can be largely resolved if the $SU(2)_L$ and $SU(3)_c$ gaugino masses satisfy the asymptotic relation $M_2/M_3 \approx 3.3$.

In addition to the Higgs discovery, the ATLAS and CMS experiments have reported an excess in Higgs production and decay in the diphoton channel, around 1.4 – 2 times larger than the SM expectations. The statistical significance of this apparent deviation from the SM prediction is at present not sufficiently strong to draw a definite conclusion, but if confirmed in the future, it will be clear indication of new physics around the electroweak scale. The charged superparticles in the MSSM can give a sizable contribution to the Higgs coupling to photons provided they are sufficiently light. A light third generation sfermions with large left-right mixing have been considered to achieve enhancement in Higgs production and decay in the diphoton channel, relative to the SM [13, 14]. In this present paper we estimate the diphoton rate $R_{\gamma\gamma}$ for the parameter space which provides a resolution of the little hierarchy problem.

The layout of this paper is as follows. In Section 2 we briefly summarize the little hierarchy problem in the MSSM. In Section 3 we briefly describe the fine-tuning conditions at low and high scales. The $SU(4)_c \times SU(2)_L \times SU(2)_R$ model we study in this paper is described in Section 4. Section 5 encapsulates the scanning procedure and the experimental constraints that we employ. Our results are discussed in Section 6 and our conclusion are presented in Section 7.

2 Little Hierarchy Problem in the MSSM

At tree level the lightest CP-even (SM-like) Higgs boson mass m_h in the MSSM is bounded from above by the mass of the Z boson

$$m_h < M_Z. \quad (1)$$

It is clear from Eq.(1) that significant radiative corrections are needed in order to accommodate values of $m_h \sim 125$ GeV. For simplicity, we show the dominant one-loop corrections to the Higgs boson mass [15]

$$m_h^2 \simeq M_Z^2 \cos^2 2\beta \left(1 - \frac{3}{8\pi^2} \frac{m_t^2}{v^2} t \right) + \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \left[t + \frac{1}{2} X_t \right], \quad (2)$$

where

$$t = \log \left(\frac{M_S^2}{M_t^2} \right), \quad X_t = \frac{2\tilde{A}_t^2}{M_S^2} \left(1 - \frac{\tilde{A}_t^2}{12M_S^2} \right), \quad \tilde{A}_t = A_t - \mu \cot \beta. \quad (3)$$

Here A_t is the trilinear soft supersymmetry breaking (SSB) parameter associated with the top quark Yukawa coupling, μ is the MSSM Higgs bilinear mixing term, $\cot \beta$ is the ratio of down and up-type Higgs vacuum expectation values (VEVs) and $M_S = \sqrt{m_{Q_t}^2 m_{U_t}^2}$ is the geometric mean of left and right stop masses squared. A 125 GeV Higgs mass requires either a very large, $\mathcal{O}(\text{few} - 10)$ TeV, stop quark mass [3], or a large SSB trilinear A_t -term, with a stop quark mass of around a TeV [4].

In the MSSM, through minimizing the tree level scalar potential, the Z boson mass $M_Z = 91.2$ GeV can be computed in terms of μ and the soft supersymmetry breaking mass terms for the up (m_{H_u}) and down (m_{H_d})-type Higgs doublets [16]

$$\frac{1}{2} M_Z^2 = -\mu^2 + \left(\frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} \right) \simeq -\mu^2 - m_{H_u}^2. \quad (4)$$

The approximation in Eq. (4) works well for moderate and large $\tan \beta$ values. We see from Eq. (4) that unless μ and m_{H_u} values are of order M_Z some fine-tuning of these

two parameters is required. As mentioned above, a 125 GeV Higgs in SUSY requires the stop quark mass to be around a TeV. Since H_u couples to the top quark, the heavy stop quark significantly affects the values of m_{H_u} and pushes it to be order TeV at either the low or GUT scale, or at both scales simultaneously. We will show below that the heavy stop quark contribution to m_{H_u} can be canceled by a suitable choice of gaugino masses at the GUT scale and, in this way, ameliorate the little hierarchy problem.

Before discussing the effects of specific choices of non-universal gaugino masses on the little hierarchy problem, we present a more general analysis with arbitrary gaugino masses at the GUT scale. For this purpose we use a semi-analytic calculation for the MSSM sparticle spectra presented in ref. [10].

$$m_{H_u}^2 \approx -2.67M_3^2 + 0.2M_2^2 - 0.091m_0^2 - 0.1A_{t_0}^2 - 0.22M_3M_2 + \dots, \quad (5)$$

$$m_{Q_t}^2 \approx 5.41M_3^2 + 0.392M_2^2 + 0.64m_0^2 + 0.115M_3A_{t_0} - 0.072M_3M_2 + \dots, \quad (6)$$

$$m_{U_t}^2 \approx 4.52M_3^2 - 0.188M_2^2 + 0.273m_0^2 - 0.066A_{t_0}^2 - 0.145M_3M_2 + \dots, \quad (7)$$

$$A_t \approx -2.012M_3 - 0.252M_2 + 0.273A_{t_0} + \dots \quad (8)$$

The following GUT scale boundary conditions are employed

$$\alpha_G = 1/24.32, \quad M_G = 2.0 \times 10^{16} \text{GeV}, \quad y_t(M_G) = 0.512 \quad (9)$$

Here $\alpha_2 = \alpha_1 = \alpha_G$ and we do not enforce exact unification $\alpha_3 = \alpha_2 = \alpha_1$ at the GUT scale, since a few percent deviation from the unification condition can be expected due to unknown GUT scale threshold corrections [17]. M_G and y_t stand for GUT scale and top Yukawa coupling. By integrating the one loop RGEs [18], the MSSM sparticle masses at M_Z scale can be expressed in terms of the GUT scale fundamental parameters $m_0, M_{1,2,3}, A_{t_0}$ and the μ term. Here m_0 stands for universal SSB mass term for sfermions, $M_{1,2,3}$ are SSB gaugino masses and A_{t_0} is the trilinear SSB coupling.

The ellipses denote additional terms which do not give significant contribution to the given sparticle masses. In the results presented later, which are based on the numerical calculation employed by ISAJET 7.84 [12], full two loop RGE's are used to calculate the parameters given in Eqs. (5)-(8).

We observe from Eq.(5) that the simplest way to reduce the absolute value of $m_{H_u}^2$ (which, for our purpose, can be regarded as a measure of the fine-tuning) is to have comparable values for the first two terms. This suggests the need for non-universal gaugino masses at M_G , with $|M_2| > |M_3|$. At the same time, from Eqs. (6) and (7) we see that it is possible to have large values of the stop quark mass.

It is interesting to note that the relative signs of M_3 and M_2 play an important role. It follows from Eq.(8) that opposite signs for M_2 and M_3 reduce the absolute value of A_t , after RGE running from high to low scale. This, however, reduces m_h and to compensate for this reduction one should increase the value of M_3 at M_{GUT} . This, in

turn, increases the absolute value of $m_{H_u}^2$ at M_{SUSY} . Thus, from Eq. (5) we find that in order to reduce the amount of fine-tuning, the following condition should be met,

$$M_2/M_3 \approx \sqrt{2.67/0.2} \approx 3.6. \quad (10)$$

The results in Eq. (10) are obtained using one loop RGE's, but as we will see later on, the ratio $M_2/M_3 \approx 3.6$ is in good agreement with our finding $M_2/M_3 \approx 3.3$ using the ISAJET 7.84 package which employs two loop RGE's.

3 Fine-Tuning Constraints

The latest (7.84) version of ISAJET [12] calculates the fine-tuning conditions related to the little hierarchy problem at M_{EW} and at the GUT scale (M_{HS}). We will briefly describe these parameters in this section.

After including the one-loop effective potential contributions to the tree level MSSM Higgs potential, M_Z the Z boson mass is given by the following relation:

$$\frac{M_Z^2}{2} = \frac{(m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2. \quad (11)$$

The Σ 's stand for the contributions coming from the one-loop effective potential (For more details see ref. [7]). All parameters in Eq. (11) are defined at the weak scale M_{EW} .

3.1 Electroweak Scale Fine-Tuning

In order to measure the EW scale fine-tuning condition associated with the little hierarchy problem, the following definitions are used [7]:

$$C_{H_d} \equiv |m_{H_d}^2/(\tan^2 \beta - 1)|, \quad C_{H_u} \equiv |-m_{H_u}^2 \tan^2 \beta/(\tan^2 \beta - 1)|, \quad C_\mu \equiv |-\mu^2|, \quad (12)$$

with each $C_{\Sigma_{u,d}^{u,d}(i)}$ less than some characteristic value of order M_Z^2 . Here, i labels the SM and supersymmetric particles that contribute to the one-loop Higgs potential. For the fine-tuning condition we have

$$\Delta_{EW} \equiv \max(C_i)/(M_Z^2/2). \quad (13)$$

Note that Eq. (13) defines the fine-tuning condition at M_{EW} without addressing the question of the origin of the parameters that are involved.

3.2 High Scale Fine-Tuning

In most SUSY breaking scenarios the parameters in Eq. (11) are defined at a scale higher than M_{EW} . In order to fully address the fine-tuning condition we need to check the relations among the parameters involved in Eq. (11) at high scale. We relate the parameters at low and high scales as follows:

$$m_{H_{u,d}}^2 = m_{H_{u,d}}^2(M_{HS}) + \delta m_{H_{u,d}}^2, \quad \mu^2 = \mu^2(M_{HS}) + \delta \mu^2. \quad (14)$$

Here $m_{H_{u,d}}^2(M_{HS})$ and $\mu^2(M_{HS})$ are the corresponding parameters renormalized at the high scale, and $\delta m_{H_{u,d}}^2$, $\delta \mu^2$ measure how the given parameter is changed due to renormalization group evolution (RGE). Eq. (11) can be re-expressed in the form

$$\begin{aligned} \frac{m_Z^2}{2} &= \frac{(m_{H_d}^2(M_{HS}) + \delta m_{H_d}^2 + \Sigma_d^d) - (m_{H_u}^2(M_{HS}) + \delta m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} \\ &- (\mu^2(M_{HS}) + \delta \mu^2). \end{aligned} \quad (15)$$

Following ref. [7], we introduce the following parameters:

$$\begin{aligned} B_{H_d} &\equiv |m_{H_d}^2(M_{HS})/(\tan^2 \beta - 1)|, B_{\delta H_d} \equiv |\delta m_{H_d}^2/(\tan^2 \beta - 1)|, \\ B_{H_u} &\equiv |-m_{H_u}^2(M_{HS}) \tan^2 \beta/(\tan^2 \beta - 1)|, B_{\mu} \equiv |\mu^2(M_{HS})|, \\ B_{\delta H_u} &\equiv |-\delta m_{H_u}^2 \tan^2 \beta/(\tan^2 \beta - 1)|, B_{\delta \mu} \equiv |\delta \mu^2|, \end{aligned} \quad (16)$$

and the high scale fine-tuning measure Δ_{HS} is defined to be

$$\Delta_{HS} \equiv \max(B_i)/(M_Z^2/2). \quad (17)$$

The current experimental bound on the chargino mass ($m_{\widetilde{W}} > 103$ GeV) [19] indicates that either Δ_{EW} or Δ_{HS} cannot be less than 1. The quantities Δ_{EW} and Δ_{HS} measure the sensitivity of the Z-boson mass to the parameters defined in Eqs. (12) and (16), such that $(100/\Delta_{EW})\%$ ($(100/\Delta_{HS})\%$) is the degree of fine-tuning at the corresponding scale.

4 The Model

In Section 6 we will show that the little hierarchy problem is largely resolved if the MSSM is embedded in the 4-2-2 model [11]. It seems natural to assume that in 4-2-2 the asymptotic (high/GUT scale) gaugino masses associated with $SU(4)_c$, $SU(2)_L$, and $SU(2)_R$ are three independent parameters. This can be reduced to two independent parameters in the presence of C parity [20], and we can have the following asymptotic relation among the MSSM gaugino masses:

$$M_1 = \frac{2}{5}M_3 + \frac{3}{5}M_2, \quad (18)$$

where M_3, M_2, M_1 denote the $SU(3)_c, SU(2)_L$ and $U(1)_Y$ asymptotic gaugino masses respectively. Reducing the parameters this way by keeping M_2 and M_3 independent while calculating M_1 using Eq. (18) in the gaugino sector is also phenomenologically motivated. The reason for this is that except for the bino, all remaining sparticle masses weakly depend on the value of M_1 , especially if the neutralino is the lightest supersymmetric particle (LSP). We will assume that due to C-parity the SSB mass terms induced at the high scale through gravity mediated supersymmetry breaking [21] are equal in magnitude for the squarks and sleptons within the family. For simplicity and for comparison with the CMSSM case, we also assume $m_0 = m_{H_u} = m_{H_d}$. Thus, we have the following fundamental parameters in the SSB sector:

$$m_0, m_{H_u}, m_{H_d}, M_2, M_3, A_0, \tan \beta. \quad (19)$$

In this paper we mainly focus on $\mu > 0$. Although not required, we will assume that the gauge coupling unification condition $g_3 = g_1 = g_2$ holds at M_{GUT} in 4-2-2. Such a scenario can arise, for example, from a higher dimensional $SO(10)$ [22] or $SU(8)$ [23] model after suitable compactification.

5 Scanning Procedure and Phenomenological Constraints

We employ the ISAJET 7.84 package [12] to perform random scans over the fundamental parameter space. In this package, the weak scale values of gauge and third generation Yukawa couplings are evolved to the GUT scale via the MSSM renormalization group equations (RGEs) in the \overline{DR} regularization scheme. We do not strictly enforce the unification condition at M_{GUT} , since a few percent deviation from unification can be assigned to unknown GUT-scale threshold corrections [17]. The deviation between $g_1 = g_2$ and g_3 at M_{GUT} is no worse than 3 – 4%. For simplicity, we do not include the Dirac neutrino Yukawa coupling in the RGEs. For possible values of this coupling, the impact on our calculations is not significant.

The various boundary conditions are imposed at the GUT scale and all the SSB parameters, along with the gauge and Yukawa couplings, are evolved back to the weak scale M_Z . In the evolution of Yukawa couplings the SUSY threshold corrections [24] are taken into account at the common scale $M_{\text{SUSY}} = \sqrt{m_{\tilde{t}_L} m_{\tilde{t}_R}}$, where $m_{\tilde{t}_L}$ and $m_{\tilde{t}_R}$ denote the masses of the third generation left and right-handed stop quarks. The entire parameter set is iteratively run between M_Z and M_{GUT} using the full 2-loop RGEs until a stable solution is obtained. To better account for leading-log corrections, one-loop step-beta functions are adopted for the gauge and Yukawa couplings, and the SSB parameters m_i are extracted from RGEs at multiple scales $m_i = m_i(m_i)$. The RGE-improved 1-loop effective potential is minimized at M_{SUSY} , which effectively accounts

for the leading 2-loop corrections. Full 1-loop radiative corrections are incorporated for all sparticle masses.

We have performed random scans for the following range of the parameter space:

$$\begin{aligned}
0 \leq m_0 = m_{H_u} = m_{H_d} &\leq 20 \text{ TeV} \\
0 \leq M_3 &\leq 5 \text{ TeV} \\
0 \leq M_2 &\leq 5 \text{ TeV} \\
2 \leq \tan \beta &\leq 60 \\
-3 \leq A_0/m_0 &\leq 3 \\
\mu &> 0
\end{aligned} \tag{20}$$

We set $m_t = 173.3 \text{ GeV}$ [25] and $m_b(m_Z) = 2.83 \text{ GeV}$, which is hard-coded into ISAJET.

In performing the random scan a uniform and logarithmic distribution of random points is first generated in the parameter space given in Eq. (20). The function RNORMX [26] is then employed to generate a gaussian distribution around each point in the parameter space. The collected data points all satisfy the requirement of REWSB with one of the neutralino being the lightest supersymmetric particle (LSP).

After collecting the data, we impose the mass bounds on all the particles [19] and use the IsaTools package [27] to implement the various phenomenological constraints. We successively apply the following experimental constraints on the data that we acquire from ISAJET:

$$\begin{aligned}
1.7 \times 10^{-9} \leq BR(B_s \rightarrow \mu^+ \mu^-) &\leq 4.7 \times 10^{-9} & [28] \\
2.85 \times 10^{-4} \leq BR(b \rightarrow s \gamma) &\leq 4.24 \times 10^{-4} (2\sigma) & [29] \\
0.15 \leq \frac{BR(B_u \rightarrow \tau \nu_\tau)_{\text{MSSM}}}{BR(B_u \rightarrow \tau \nu_\tau)_{\text{SM}}} &\leq 2.41 (3\sigma) & [29] \\
0 \leq \Delta(g-2)_\mu/2 &\leq 55.6 \times 10^{-10} & [30]
\end{aligned}$$

Note that for $\Delta(g-2)_\mu$, we only require that the model does no worse than the SM.

6 Results

In Figures 1, 2 and 3 we compare the CMSSM with the 4-2-2 model. Figure 1 shows our results in the $\Delta_{EW} - \Delta_{HS}$ planes for the CMSSM and 4-2-2 models. The gray points are consistent with REWSB and LSP neutralino. The orange points, a subset of the gray ones, satisfy all the constraints described in Section 5. The green points correspond to the light CP-even Higgs mass range

$$123 \text{ GeV} \leq m_h \leq 127 \text{ GeV}, \tag{21}$$

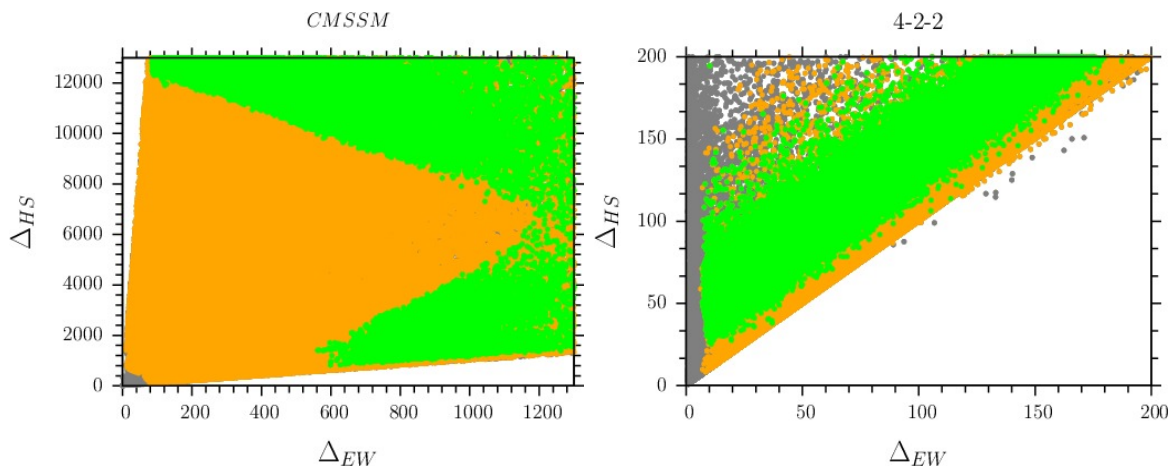


Figure 1: Plots in the $\Delta_{HS} - \Delta_{EW}$ planes for CMSSM and 4-2-2 cases. Gray points are consistent with REWSB and neutralino to be LSP. The orange points form a subset of the gray ones and satisfy all the constraints described in Section 5. Green points belong to the subset of orange points and satisfy the Higgs mass range $123 \text{ GeV} \leq m_h \leq 127 \text{ GeV}$.

and form a subset of the orange points. We assume that the Higgs boson mass range in Eq. (21) is compatible with the current experimental bound, while allowing for a 2-3 GeV error margin in the RG-improved one-loop effective potential calculation [31]. Note that varying the top quark mass within a 1σ interval can increase the Higgs boson mass by 1 GeV or so [32].

We do not apply the WMAP bound [33] on the relic LSP abundance since the neutralino is mostly a higgsino-like particle for the natural SUSY case. This usually yields small values for the thermal LSP relic abundance, unless the LSP neutralino mass is around 1 TeV or so [34]. In ref. [35] the axion mechanism has been invoked to yield the desired relic abundance for axion-nonthermal LSP dark matter.

The green points in the $\Delta_{EW} - \Delta_{HS}$ plane for CMSSM indicate that it is possible to have $\Delta_{EW} \approx 50$ (green points upper left corner), but with $\Delta_{HS} > 12,000$ (0.008%). On the other hand in the CMSSM for minimal value of Δ_{HS} , namely $\Delta_{HS} \approx 800$ (0.12%) (fine-tuning), we also find a minimal value for $\Delta_{EW} \approx 600$ (0.17%). This observation shows how important it is to simultaneously study both the low and high scale fine-tuning conditions for the little hierarchy problem in order to have the complete picture.

We now focus on the 4-2-2 model, which does much better than the CMSSM in terms of the little hierarchy problem. We find that for 4-2-2 it is possible to have solutions with much smaller values for Δ_{EW} and Δ_{HS} that are consistent with all collider constraints described in Section 5, and with the light CP even Higgs having mass in the interval presented in Eq. (21). Thus, Δ_{EW} can be as small as 10, with

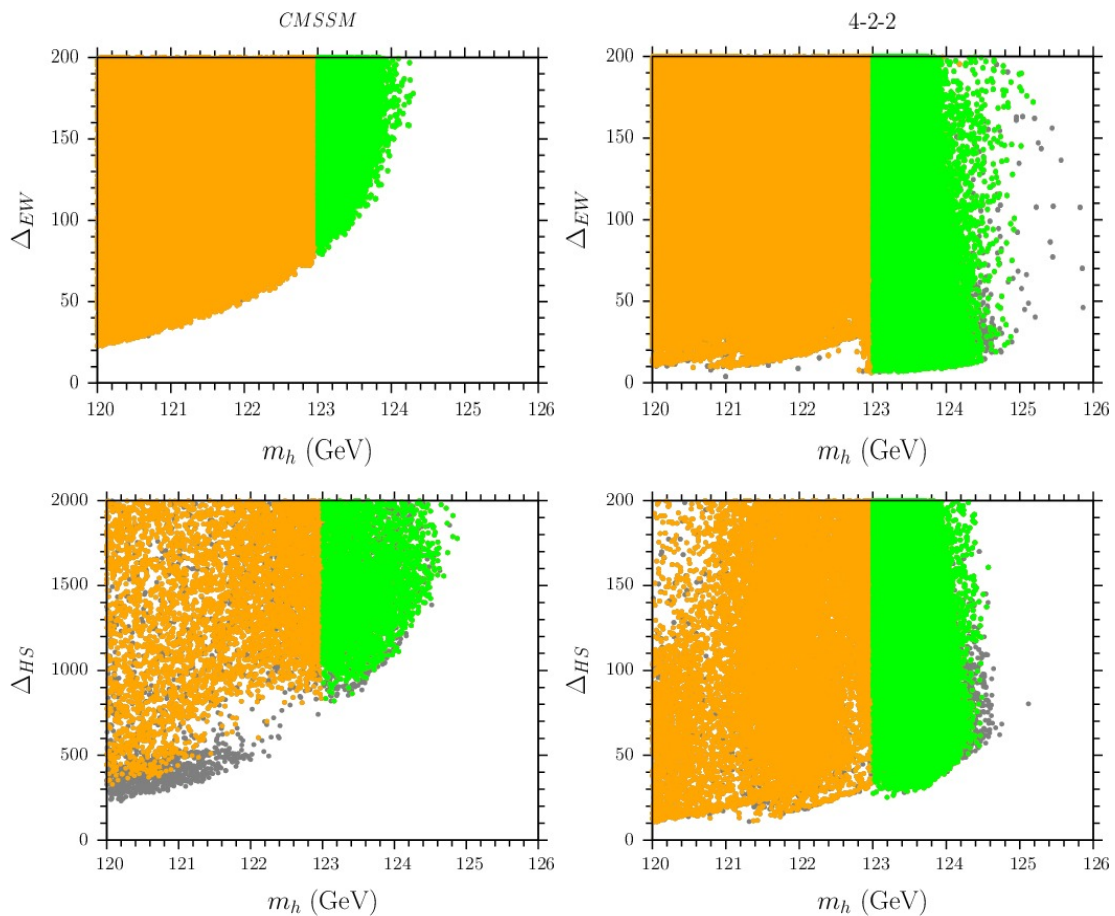


Figure 2: Plots in $\Delta_{EW} - m_h$ and $\Delta_{HS} - m_h$ planes for CMSSM and 4-2-2. Color coding is the same as described in Figure 1.

$\Delta_{HS} \approx 20$.

In Figure 2 we present the results in $\Delta_{EW} - m_h$ and $\Delta_{HS} - m_h$ planes for the CMSSM and 4-2-2 cases. The color coding is the same as in Figure 1. Green points in the $\Delta_{EW} - m_h$ plane can be as low as 80 for CMSSM, but as we learned from Figure 1, these points correspond to $\Delta_{HS} > 12,000$. Note that the fine-tuning parameters Δ_{EW} and Δ_{HS} in CMSSM rapidly change with changes in the light CP even Higgs boson mass, especially when $m_h > 123$ GeV. In contrast, the 4-2-2 model shows a very mild dependence of Δ_{EW} and Δ_{HS} on the Higgs boson mass up to the 125 GeV.

In Figure 3 we show how the fine-tuning parameters Δ_{EW} and Δ_{HS} differ in their dependence on the SSB mass parameter m_0 . We present both the CMSSM and 4-2-2 cases in this figure. For CMSSM, the Δ_{EW} vs. m_0 plane shows that minimal values for Δ_{EW} correspond to $m_0 \approx 7$ TeV. On the other hand, we see from the $\Delta_{HS} - m_0$

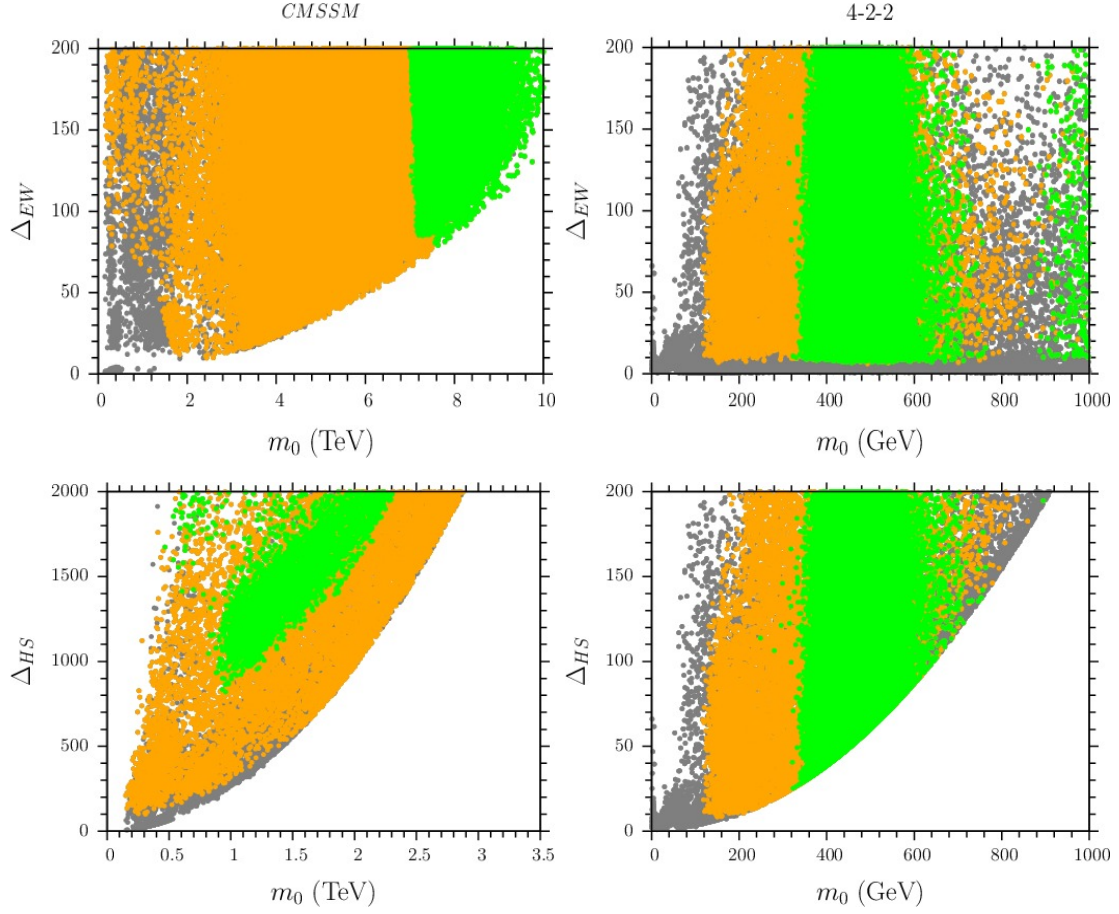


Figure 3: Plots in $\Delta_{EW} - m_0$ and $\Delta_{HS} - m_0$ planes for CMSSM and 4-2-2 cases. Color coding is the same as described in Figure 1.

figure that the minimal values for Δ_{HS} are achieved for $m_0 \approx 1$ TeV.

We can observe a different relationship among these parameters in the $\Delta_{EW} - m_0$ and $\Delta_{HS} - m_0$ planes in Figure 3 for the 4-2-2 case. In the $\Delta_{EW} - m_0$ plane we see that Δ_{EW} depends very mildly on the values of m_0 , for $0 < m_0 < 1$ TeV. The $\Delta_{HS} - m_0$ plane shows that Δ_{HS} varies significantly as m_0 increases. We also observe from Figure 3 that in the 4-2-2 case, m_0 can be as light as 400 GeV with $m_h > 123$ GeV and relatively small Δ_{EW} and Δ_{HS} values. This finding can be understood from Figure 4 where results are presented in $M_3 - M_2$ and $m_0 - M_3$ planes.

In Figure 4 the gray points are consistent with REWSB and LSP neutralino. The orange points form a subset of the gray ones and satisfy all the constraints described in Section 5. The purple points form a subset of the orange points with $\Delta_{EW} \leq 100$ and $\Delta_{HS} \leq 100$ conditions. The green points belong to a subset of purple points and

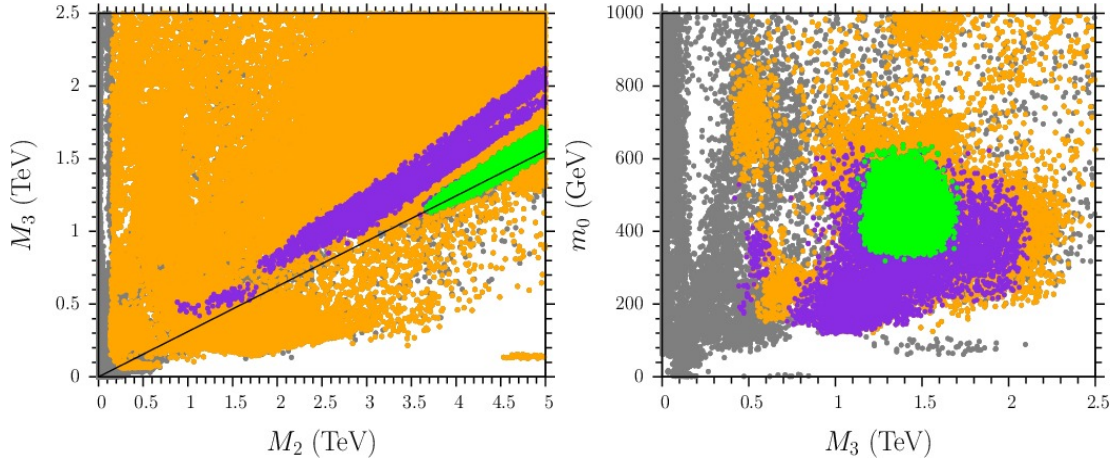


Figure 4: Plots in the $M_3 - M_2$ and $m_0 - M_3$ planes for 4-2-2. Gray points are consistent with REWSB and neutralino LSP. The orange points, a subset of the gray ones, satisfy all the constraints described in Section 5. Purple points are subset of orange points with $\Delta_{EW} \leq 100$ and $\Delta_{HS} \leq 100$. Green points belong to a subset of purple points with the Higgs mass range $123 \text{ GeV} \leq m_h \leq 127 \text{ GeV}$.

satisfy the Higgs mass range $123 \text{ GeV} \leq m_h \leq 127 \text{ GeV}$.

From the $M_3 - M_2$ plane we can see that some green points lie near the line corresponding to $M_2/M_3 \approx 3.3$. As discussed in Section 2, in order to have a cancellation between M_2 and M_3 in the expression for M_{H_u} , the ratio needs to satisfy $M_2/M_3 \approx 3.6$. This result was obtained using one-loop RGE's for the SSB parameters (see Eqs. (5)-(7)), but still serves as a good approximation to describe the green points in $M_3 - M_2$ plane. At the same time we can see from Eqs. (6) and (7) that having $M_2/M_3 \approx 3.6$ does not cancel the M_3 and M_2 contributions in the expressions for the stop quark, and so the stop quark mass can still be large. As we know, a large stop quark mass is required to provide significant radiative contributions that yield a light CP-even Higgs mass of around 125 GeV.

We show that the interval $400 \text{ GeV} < m_0 < 600 \text{ GeV}$ and $1.1 \text{ TeV} < M_3 < 1.7 \text{ TeV}$ in 4-2-2 model is in good agreement with the current experimental bounds.

In Figure 5 we show how the parameter space changes if solution of the little hierarchy problem is required at the low or high scale. We present the results in $\Delta_{EW} - A_0/m_0$, $\Delta_{HS} - A_0/m_0$, $\Delta_{EW} - \tan \beta$ and $\Delta_{HS} - \tan \beta$ planes. Color coding is same as in Figure 1. We see from the $\Delta_{EW} - A_0/m_0$ plane that requiring only low scale fine-tuning ($\Delta_{EW} < 100$) restricts the sign of the ratio A_0/m_0 . In other words we can have a solution to the little hierarchy problem for any negative value of A_0 . For the high scale fine-tuning case ($\Delta_{HS} < 100$), we have a relatively narrow interval $-3 < A_0/m_0 < -1.5$. A similar situation occurs for the $\tan \beta$ case, where the allowed

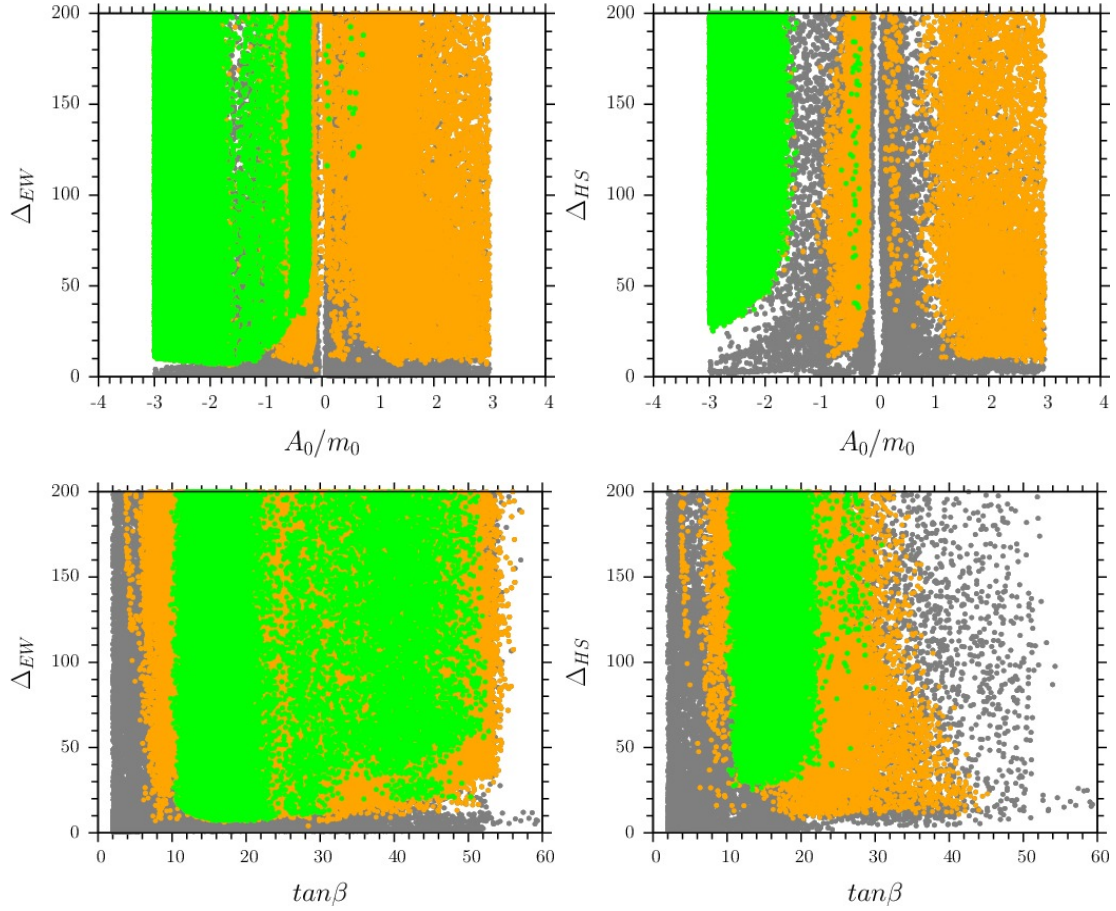


Figure 5: Plots in $\Delta_{EW} - A_0/m_0$, $\Delta_{HS} - A_0/m_0$, $\Delta_{EW} - \tan\beta$, $\Delta_{HS} - \tan\beta$ planes for 4-2-2. Color coding is the same as described in Figure 1.

interval for $\tan\beta$ changes significantly when we compare low scale fine-tuning with the high scale one. The $\Delta_{EW} - \tan\beta$ plane indicates that there is no little hierarchy problem for $10 < \tan\beta < 55$. On the other hand, from the $\Delta_{HS} - \tan\beta$ plane we see that the range $10 < \tan\beta < 22$ has $\Delta_{HS} < 100$ and $123 \text{ GeV} \leq m_h \leq 127 \text{ GeV}$.

Note that this also shows that precise $t - b - \tau$ Yukawa unification is not compatible with the 4-2-2 model if we insist on the resolution of the little hierarchy problem. This is because, the solution of the little hierarchy problem prefers $\tan\beta < 20$, while precise $t - b - \tau$ Yukawa unification predicts $\tan\beta \approx 50$ [36].

In Figure 6, we present the SUSY particle spectrum in the $m_{\tilde{q}} - m_{\tilde{g}}$, $m_{\tilde{t}_1} - m_{\tilde{\chi}_1^0}$, $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0}$ planes. Color coding is the same as described in Figure 1. In addition, we have used maroon color to denote a subset of the green points, that have $\Delta_{HS} < 100$ and $\Delta_{EW} < 100$. The $m_{\tilde{q}} - m_{\tilde{g}}$ panel shows that in the 4-2-2 model,

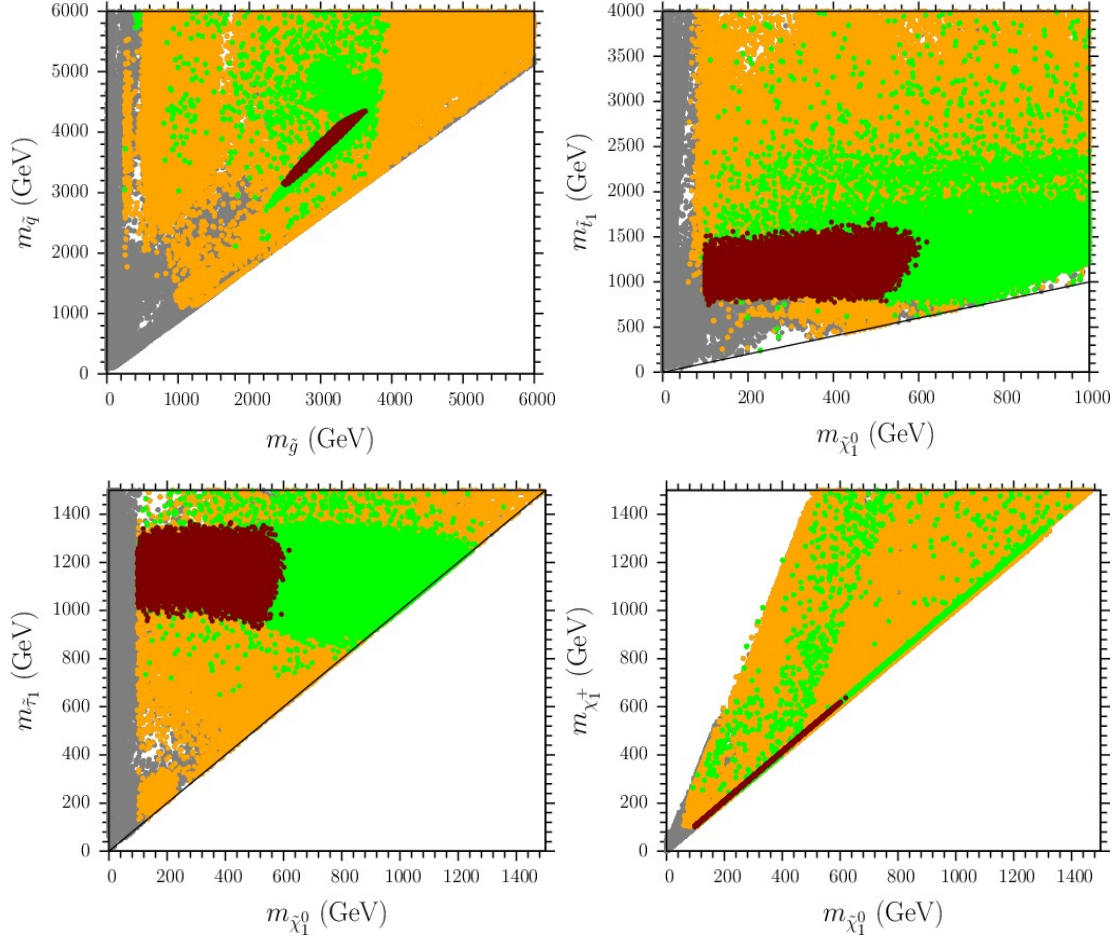


Figure 6: Plots in the $m_{\tilde{q}} - m_{\tilde{g}}$, $m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0}$, $m_{\tilde{t}_1} - m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}$ planes for 4-2-2. Color coding is the same as described in Figure 1. In addition, we have used maroon color to denote a subset of the green points with $\Delta_{HS} < 100$ and $\Delta_{EW} < 100$.

resolution of the little hierarchy problem with acceptable Higgs boson mass yields a gluino mass in the interval $2800 \text{ GeV} < m_{\tilde{g}} < 4000 \text{ GeV}$ and the first two family squark masses in the range $3000 \text{ GeV} < m_{\tilde{q}} < 4200 \text{ GeV}$. These intervals are fully compatible with the lower mass bounds obtained by the ATLAS and CMS collaborations.

It is possible in a supersymmetric theory to have a stop quark or a stau lepton as the next to lightest supersymmetric particle (NLSP). For such a scenario, we can obtain the desired LSP relic abundance in the universe through co-annihilation. However, we see from the $m_{\tilde{t}_1} - m_{\tilde{\chi}_1^0}$, $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}$ panels that the maroon points are far from the unit line. This indicates that in the 4-2-2 model we cannot have neutralino-stop (or stau) co-annihilation scenario. Therefore, the dark matter relic abundance required by

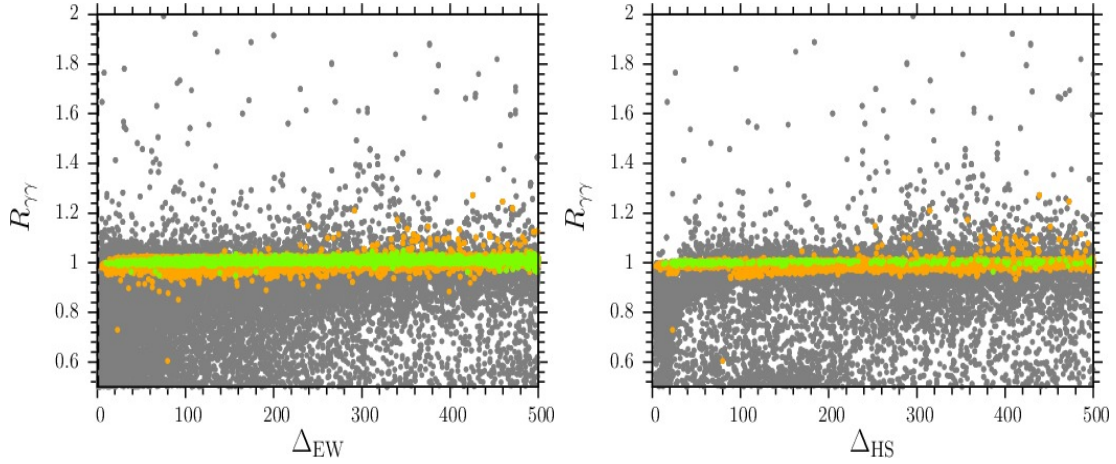


Figure 7: Plots in $R_{\gamma\gamma} - \Delta_{EW}$, and $R_{\gamma\gamma} - \Delta_{HS}$ planes for 4-2-2 case. Color coding is the same as described in Figure 1.

WMAP is not compatible in the 4-2-2 model with the resolution of the little hierarchy problem.

In the $m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0}$ plane we observe that the maroon points lie near the unit line, which indicates that the lightest neutralino is mostly higgsino-like. This yields relatively low values for relic abundance unless the LSP neutralino mass is around 1 TeV [34]. If we relax the condition for natural SUSY and look for solutions with $\Delta_{EW} < 150$ and $\Delta_{HS} < 150$, the higgsino-like LSP neutralino can be the desired dark matter candidate.

Motivated by the recent results from Higgs searches at the LHC, we also calculate the Higgs production and decay in the diphoton channel for the 4-2-2 model. The particle spectrum was calculated using the ISAJET 7.84 package [12] and this was interfaced with FeynHiggs 2.9.4 [37] to estimate the production cross section and decay width. We introduce a parameter $R_{\gamma\gamma}$ to quantify possible excess in Higgs production and decay in the diphoton channel over the Standard Model expectation,

$$R_{\gamma\gamma} \equiv \frac{\sigma(h) \times Br(h \rightarrow \gamma\gamma)}{(\sigma(h) \times Br(h \rightarrow \gamma\gamma))_{SM}}. \quad (22)$$

In Figure 7 we show the results in $R_{\gamma\gamma} - \Delta_{EW}$ and $R_{\gamma\gamma} - \Delta_{HS}$ planes for the 4-2-2 model. Color coding is the same as described in Figure 1. We did not find any solution in the natural SUSY limit corresponding to $R_{\gamma\gamma} > 1.2$. In the $R_{\gamma\gamma} - \Delta_{EW}$ plane there are some orange points with $R_{\gamma\gamma} > 1.2$ with Δ_{EW} and Δ_{HS} both larger than 400, but these solution do not satisfy the Higgs mass bound (green points). As shown, for instance, in ref. [14], in order to have $R_{\gamma\gamma} > 1.2$ in the MSSM it is necessary to have either a stop or stau around 100 GeV. This condition is not sufficient but, for this

	Point 1	Point 2	Point 3
m_0	1086.39	460.72	497.64
M_1	979.1	3313.58	3606.12
M_2	979.1	4579.38	4908.89
M_3	979.1	1414.89	1651.94
A_0	-3244.79	-1270.88	-1390.14
$\tan \beta$	28.49	15.41	16.47
μ	1853	176	746
m_h	124.06	124	124.1
m_H	1862	2856	3109
m_A	1850	2838	3088
m_{H^\pm}	1864	2857	3110
$m_{\tilde{\chi}_{1,2}^0}$	424, 807	180, 182	759, 762
$m_{\tilde{\chi}_{3,4}^0}$	1845, 1847	1477, 3757	1620, 4032
$m_{\tilde{\chi}_{1,2}^\pm}$	810, 1850	188, 3754	780, 4023
$m_{\tilde{g}}$	2180	3048	3515
$m_{\tilde{u}_{L,R}}$	2239, 2174	3842, 2719	4253, 3118
$m_{\tilde{t}_{1,2}}$	1084, 1744	1039, 3394	1467, 3768
$m_{\tilde{d}_{L,R}}$	2240, 2166	3843, 2629	4254, 3025
$m_{\tilde{b}_{1,2}}$	1721, 1947	2524, 3436	2905, 3808
$m_{\tilde{\nu}_1}$	1261	2980	3182
$m_{\tilde{\nu}_3}$	1098	2972	3164
$m_{\tilde{e}_{L,R}}$	1265, 1144	2978, 1296	3181, 1407
$m_{\tilde{\tau}_{1,2}}$	719, 1107	1189, 2961	1276, 3156
$\sigma_{SI}(\text{pb})$	9.24×10^{-12}	1.79×10^{-10}	2.84×10^{-10}
$\sigma_{SD}(\text{pb})$	2.46×10^{-09}	2.29×10^{-06}	2.36×10^{-07}
$\Omega_{CDM}h^2$	7.06	0.007	0.11
Δ_{EW}	827	15.4	134
Δ_{HS}	1110	51.3	181

Table 1: Point 1 displays solution with minimal value of Δ_{EW} and Δ_{HS} in the framework of CMSSM. Point 2 represents minimal value of Δ_{EW} and Δ_{HS} in the 4-2-2 model. Point 3 depict solutions corresponding minimal Δ_{EW} and Δ_{HS} and best $\Omega_{CDM}h^2$ values. All benchmark points satisfying the various constraints mentioned in Section 5

discussion, we will restrict ourselves to the necessary condition. In Figure 6 from the $m_{\tilde{t}_1} - m_{\tilde{\chi}_1^0}$ and $m_{\tilde{\tau}_1} - m_{\tilde{\chi}_1^0}$ planes we observe that the Higgs mass bound (green points) requires minimal values for the stop and stau to be larger than 500 GeV. This explains

why we do not find solutions with $R_{\gamma\gamma} > 1.2$ in this model.

In Table 1, we show three benchmark points satisfying the various constraints mentioned in Section 5. These display the minimal values of Δ_{EW} and Δ_{HS} that are compatible with 124 GeV CP-even Higgs boson. Point 1 displays solution with minimal value of Δ_{EW} and Δ_{HS} in the framework of CMSSM. Point 2 represents minimal value of Δ_{EW} and Δ_{HS} in the 4-2-2 model. Point 3 depict solutions corresponding minimal Δ_{EW} and Δ_{HS} and best $\Omega_{CDM}h^2$ values.

7 Conclusion

By imposing conditions for natural SUSY ($\Delta_{EW} < 100$ and $\Delta_{HS} < 100$) and requiring $123 \text{ GeV} < m_h < 127 \text{ GeV}$, we obtain a distinctive particle spectra characterized by relatively light third generation sfermions. The light stop quark mass lies in the range $700 \text{ GeV} < m_{\tilde{t}_1} < 1500 \text{ GeV}$, and the range for the light stau lepton mass is $900 \text{ GeV} < m_{\tilde{\tau}_1} < 1300 \text{ GeV}$. The first two family squarks lie in the mass range $3000 \text{ GeV} < m_{\tilde{t}_1} < 4500 \text{ GeV}$, and for the gluino we find $2500 \text{ GeV} < m_{\tilde{g}_1} < 3500 \text{ GeV}$. It is interesting that the bound for $\tan\beta$ is $10 < \tan\beta < 20$. Although the higgsino-like chargino and neutralino can be $O(100) \text{ GeV}$, was shown in ref. [38] observing it at the LHC may still be difficult. If the excess in the Higgs production and decay in the diphoton channel is confirmed in future experiments it will rule out this class of Supersymmetric $SU(4)_c \times SU(2)_L \times SU(2)_R$ model with natural Supersymmetry.

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